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**Centro di Simulazione Numerica Avanzata – CeSNA**  
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## Some recent developments in strain-gradient elasticity and their applications to fracture mechanics

Standard continuum mechanics theories fail to describe a wide class of engineering problems characterized by size-effects; this inability is due to the absence of any internal length, in the constitutive equations, aimed at modelling the underlying material microstructure.

We discuss, in particular, stress localization phenomena as occurring in fracture mechanics; therein strain-gradient elasticity can be used to describe specific interactions between the material microstructure and the fracture process zone. We have derived the asymptotic solutions for all the crack opening problems (modes I, II and III) for an elastic strain-gradient material characterised by five independent material lengths: in the stored elastic energy these lengths penalize torsion, bending, differential shear and differential elongation; a latter constant represents a coupling parameter among these four classes of deformations. The explicit forms of the energy release rates as functions of the characteristic lengths allow to discuss in some detail the strengthening effects associated to some material microstructures.

### REFERENCES

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**MS1 Conference Room,**  
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