Soundness Arguments for Consistency and Transmission Failure

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This talk presents an epistemological investigation of *Soundness Arguments for Consistency*. For a given theory S, a soundness argument for the consistency of S aims to show that S is *consistent* via a detour, that is, by first showing that S is *sound*, i.e., that all theorems of S are true. Although such arguments are virtually unanimously accepted as valid, it is unclear whether soundness arguments have any *epistemic value*, or in other words, it is unclear whether such arguments can, in principle, be employed by agents to improve their epistemic situation with respect to the consistency of S. Philosophers have mixed intuitions about the epistemic value of soundness arguments: (Girard, 1987), (Dummett, 1978) and (Wright, 1994) separately pointed out that such arguments are little epistemic value. Recently, (Piazza and Pulcini, 2013) argued that all soundness arguments are ill-founded. On the other hand, (Shapiro, 1998) and (Horsten, 2021) attribute to soundness arguments some epistemic value. However, (to my best knowledge) not much work has been done to uncover the epistemology of mathematics behind these intuitions.

This talk investigates the epistemic value of soundness arguments and focuses on the question of whether soundness arguments are *cogent*, i.e., whether soundness arguments can, in principle, be employed to acquire a justification to believe their conclusion *in virtue* of (i) the argument's premises being justified and (ii) the argument's validity. I will show that the answer to the question of cogency depends on the *superstructure* of mathematical justification, and in particular, on what kind of higher-order conditions must be in place for mathematical beliefs to be justified. I will discuss the two main positions with respect to the superstructure of justification (known from the literature on perceptual justification), Liberalism and Conservativism, and

show how these two positions imply different, incompatible answers to the question of cogency: Liberalism implies that soundness arguments are cogent, whereas conservativism implies the failure of cogency. Towards the end of the talk, if time permits, I will make some more exploratory comments to motivate the intuition that soundness arguments fail to be cogent independently from Conservativism.

References

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